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Small SUSY phases in string-inspired supergravity

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Abstract

In supersymmetric models, there are new CP violating phases which, if unsuppressed, would give a too large neutron electric dipole moment. We examine the possibility of small SUSY phases in string-inspired supergravity models in which supersymmetry is broken by the auxiliary components of the dilaton and moduli superfields. It is found that the SUSY phases can be suppressed by a small factor governing the breakdown of the approximate Peccei Quinn symmetries nonlinearly realized for the moduli superfields that participate in supersymmetry breaking. In many cases, the symmetry breaking factors are exponentially small for moderately large values of the moduli, leading to small phase values in a natural way.

It is well known that supersymmetric models have new sources of CP violation other than the QCD angle and the Kobayashi-Maskawa phase that exist already in the standard model. If these new CP-violating phases are unsuppressed and also the superpartners have masses around 100 GeV, the resulting neutron electric dipole moment would exceed the current experimental bound by a factor of $10^2 - 10^3$ [1]. To resolve this difficulty, one needs to have either the new phases smaller than $10^{-2} - 10^{-3}$ or superpartners, particularly the squarks, having masses greater than a few TeV [2]. Although the option of heavy superpartners is still possible, it is more customary to take the small phase option while keeping the superpartner masses to be around 100 GeV. Then the required smallness is somewhat disturbing, and thus it is desirable to have any explanation for the small phase values.

The above small phase problem is in fact a problem of supersymmetry (SUSY) breaking since the phases and also the superpartner masses are determined mainly by the parameters of soft SUSY breaking terms. Presently the most popular way to break SUSY is to utilize a hidden sector in the context of $N = 1$ supergravity theories [3]. Among phenomenologically acceptable supergravity models, those from superstrings are of particular interests. In string-inspired supergravity models, it is commonly assumed that supersymmetry is broken by the auxiliary components of the dilaton and/or moduli superfields whose superpotential is induced by nonperturbative hidden sector dynamics [4]. In this paper, we explore the possibility of small SUSY phases in string-inspired supergravity models adopting this scenario of SUSY breaking [5]. It is found that the SUSY phases can be suppressed by a small factor governing the breakdown of the approximate Peccei Quinn (PQ) symmetries

that are associated with the pseudoscalar components of the moduli [6]. The novel feature of this suppression mechanism is that it is completely independent of how CP is broken [7]. If the PQ symmetries for the moduli that participate in SUSY breaking are good enough, then the resulting SUSY phases would be small enough. However if a modulus whose PQ symmetry is badly broken contributes to SUSY breaking significantly, it looks quite difficult to achieve the small SUSY phases unless a strong assumption on CP violation is made.

To begin with, let us define the SUSY phase problem more precisely. We consider a low energy supersymmetric model with the superpotential

$$W = \lambda_{ijk}\Phi_i\Phi_j\Phi_k + \mu H_1 H_2, \tag{1}$$

and the soft breaking part containing

$$\frac{1}{2}m_a\lambda_a\lambda_a + A_{ijk}\varphi_i\varphi_j\varphi_k + Bh_1h_2, \tag{2}$$

where Φ_i denote generic chiral superfields with their scalar components φ_i, λ_a are gauginos, and $H_{1,2}$ ($h_{1,2}$) stand for the two Higgs superfields (their scalar components) in the minimal supersymmetric standard model. If one does not make any assumption on CP, all parameters above are complex in general. Then compared to the non-supersymmetric counterpart, the theory contains new CP-violating phases:

$$\phi_A = \{\arg(\frac{A_{ijk}}{\lambda_{ijk}})\}, \ \ \phi_B = \{\arg(\frac{B}{\mu})\}, \ \ \phi_C = \{\arg(m_a)\}. \tag{3}$$

These phases, more precisely the combinations $\phi = \{\phi_A - \phi_C, \phi_B - \phi_C\}$, give the neutron electric dipole moment $d_n \simeq (10^{-22} - 10^{-23}) \times \sin\phi$ e-cm for the

superpartner masses around 100 GeV [1]. Then the phases are constrained as $\phi \leq 10^{-2} - 10^{-3}$. Of course, as we have mentioned, one can relax this constraint by assuming that the squark masses are larger.

Let us consider a supergravity model which is assumed to enjoy the following properties of superstring vacua. First of all, the model contains a hidden sector which generally has a large gauge group as well as matter fields that transform nontrivially under the hidden sector gauge group. Also the model contains the dilaton multiplet S and the moduli multiplets which describe a variety of deformations of the internal space. Among the moduli, we consider only the overall modulus T for a moment. Later we will discuss the effects of including other moduli. The dilaton component $\text{Re}(S)$ couples to the gauge kinetic terms, giving the gauge coupling constant as $g^2 = 1/\text{Re}(S)$ at string tree level. The modulus field $\text{Re}(T)$ characterizes the size of the internal space. Then $1/\text{Re}(T)$ corresponds to the sigma model coupling constant. The imaginary components $\text{Im}(S)$ and $\text{Im}(T)$ correspond to the model-independent axion and the internal axion respectively.

As is well known, a four-dimensional $N = 1$ supergravity action is characterized by the Kähler potential K, the superpotential W, and the gauge kinetic function f_a for the a-th gauge group. At the compactification scale, one may expand the Kähler and superpotential in chiral matter fields Φ_i as

$$K = \tilde{K} + Z_{ij}\Phi_i\bar{\Phi}_j + (Y H_1 H_2 + \text{h.c.}) + ...,$$
$$W = \tilde{W} + \tilde{\mu}H_1 H_2 + \tilde{\lambda}_{ijk}\Phi_i\Phi_j\Phi_k + ..., \tag{4}$$

where all coefficients in the expansion are generic functions of S and T, and the ellipses stands for higher order terms. In perturbation theory, $\tilde{W}$ and $\tilde{\mu}$

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vanish, but they are induced by nonperturbative effects in the hidden sector which is integrated out already. Although the wavefunction factor Z_{ij} can have an off-diagonal element in general, here we assume it is diagonal, viz $Z_{ij} = Z_i \delta_{ij}$, for the sake of simplicity. At any rate, off-diagonal elements are required to be small, roughly smaller than $10^{-2} Z_i$, to avoid a too large flavor changing neutral current effect. In string theory, CP corresponds to a discrete gauge symmetry [8], and thus it must be broken spontaneously. However if broken at the compactification scale, it would appear to be explicitly broken in the $d = 4$ effective lagrangian. Here we do not make any assumption on the nature of CP violation, and thus allow all complex parameters in the Kähler and superpotential to have the phases of order unity in general [9].

Let us now assume that SUSY is broken by the auxiliary components of the dilaton S and the overall modulus T:

$$\bar{F}_I = e^{\tilde{K}/2} |\tilde{W}| (\bar{\partial}_I \partial_J \tilde{K})^{-1} (\partial_J \tilde{K} + \partial_J \ln \tilde{W}), \qquad (5)$$

where the indices $I, J = S, T$. It is then straightforward to derive the resulting global SUSY theory together with the soft breaking terms [5, 10]. If one writes the effective superpotential (for un-normalized fields) as in eq. (1),

$$\lambda_{ijk} = e^{-i\xi} e^{\tilde{K}/2} \tilde{\lambda}_{ijk}, \quad \mu = \mu_1 + \mu_2 + \mu_3, \qquad (6)$$

where $\mu_1 = \lambda \langle N \rangle$, $\mu_2 = (m_{3/2} - \bar{F}_I \bar{\partial}_I) Y$, $\mu_3 = e^{-i\xi} e^{\tilde{K}/2} \tilde{\mu}$ ($\xi = \arg(\tilde{W})$), and $m_{3/2} = e^{\tilde{K}/2} |\tilde{W}|$ denotes the gravitino mass. Note that here we consider three possible sources of the μ-term. Amongst them, the μ_1-piece is obtained by replacing the singlet field N which has the Yukawa coupling $\lambda N H_1 H_2$ by its vacuum value. If we do not have any such singlet as in the minimal

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supersymmetric standard model, then $\mu_1 = 0$. For the soft terms written as in eq. (2), one finds [10]

$$
m_a = \frac{1}{2} g_a^2 F_I \partial_I f_a,
$$
$$
A_{ijk} = \lambda_{ijk} F_I \partial_I [\ln(e^{\tilde{K}} \tilde{\lambda}_{ijk} / Z_i Z_j Z_k)],
$$
$$
B = B_1 + B_2 + B_3, \tag{7}
$$

where

$$
B_1/\mu_1 = A_\lambda/\lambda,
$$
$$
B_2/\mu_2 = F_I \partial_I [\ln(e^{\tilde{K}/2} \mu_2 / Z_{H_1} Z_{H_2})] - m_{3/2},
$$
$$
B_3/\mu_3 = F_I \partial_I [\ln(e^{\tilde{K}} \tilde{\mu} / Z_{H_1} Z_{H_2})] - m_{3/2}. \tag{8}
$$

Here A_λ denote the A-coefficient of the trilinear soft term for the term $\lambda N H_1 H_2$ in the superpotential. These soft parameters are defined at the compactification scale while the experimental constraints stand for those defined at the weak scale. For the phases $\phi_A = \arg(A_{ijk}/\lambda_{ijk})$, $\phi_B = \arg(B/\mu)$, and $\phi_C = \arg(m_a)$, this point is not so relevant since $\phi_{A,B,C}$ at the weak scale remain to be small enough as long as they are less than $10^{-2} - 10^{-3}$ at the compactification scale [11].

The above formulae for soft terms show that there are a lot of potentially complex quantities which can contribute to the phases $\phi_{A,B,C}$. First of all, the SUSY breaking order parameters F_I can be complex in general. A nonzero $\arg(F_I)$ may arise due to nonzero vacuum values of $\text{Im}(S)$ and $\text{Im}(T)$, or due to the complex Yukawa couplings of hidden matters which would affect the induced superpotential $\tilde{W}$. Furthermore, although $\tilde{K}$ and Z_i are real functions, their derivatives $\partial_I \tilde{K}$, $\partial_I \bar{\partial}_J \tilde{K}$, and $\partial_I Z_i$ can be complex. Besides

these, we can have complex $\partial_I f_a$, $\partial_I \ln(\tilde{\lambda}_{ijk})$, $\partial_I \ln(Y)$, $\partial_I \ln(\tilde{\mu})$, and several others. It is then convenient to classify all the relevant (potentially) complex quantities as follows:

$$X_1 : \partial_I \tilde{K}, \partial_I \bar{\partial}_J \tilde{K}, \partial_I Z_i, \partial_I f_a, \partial_I \ln(\tilde{\lambda}_{ijk});$$
$$X_2 : \partial_I \ln(\tilde{W}); \quad X_3 : \partial_I \ln(\tilde{\mu});$$
$$X_4 : \partial_I \ln(Y), \bar{\partial}_I \ln(Y), \partial_I \bar{\partial}_J \ln(Y). \tag{9}$$

It is easy to see that if X_1 and X_2 are all real, then ϕ_A and ϕ_C do vanish. The phase ϕ_B is affected also by X_3 and X_4, and thus making it small requires more conditions.

At first sight, it looks very nontrivial to make all the above quantites real. However as we will see, due to the approximate PQ symmetries nonlinearly realized for $\text{Im}(S)$ and $\text{Im}(T)$, many of them are in fact (approximately) real. In spacetime and world sheet perturbation theory, the vertex operators of $\text{Im}(S)$ and $\text{Im}(T)$ vanish at zero momentum [6]. Then the corresponding perturbative effective action would be invariant under the PQ symmetries:

$$U(1)_S : S \to S + i\alpha_S, \quad U(1)_T : T \to T + i\alpha_T, \tag{10}$$

where $\alpha_{S,T}$ are arbitrary real constants. The symmetry $U(1)_T$ is expected to be broken by nonperturbative effects on world sheet, i.e. world sheet instantons [12], even at string tree level while $U(1)_S$ is broken only by nonperturbative effects on spacetime. As a result, their breakdown is suppressed either by $e^{-c_1 S}$ (for $U(1)_S$) or by $e^{-c_2 T}$ (for $U(1)_T$) where c_1 and c_2 are some real constants. For S normalized as $g^2 = 1/\text{Re}(S)$ at string tree level, c_1 is of $O(4\pi^2)$. Then for the phenomenologically favored $g^2 \simeq 1$, we have

$|e^{-c_1 S}| \ll 10^{-3}$. This implies that one can safely ignore $U(1)_S$-violating effects for the discussion of the SUSY phases if they are *not* the leading effects, but just give small corrections to the leading perturbative effects. A common normalization of T is that of $T \equiv T + i$ for which the world sheet instanton factor is given by $q \equiv e^{-2\pi T}$. Then all $U(1)_T$-violating corrections are suppressed by a factor of $O(q)$.

For the supergravity action invariant under $U(1)_S$ and $U(1)_T$, the corresponding Kähler potential can be chosen to be invariant. Then the superpotential should be invariant up to a constant phase and the gauge kinetic functions up to imaginary constants [13]. This implies that (i) $\tilde{K}$ and Z_i are the real functions of the real variables $\mathrm{Re}(S)$ and $\mathrm{Re}(T)$ up to corrections of $O(q)$, (ii) the gauge kinetic functions $f_a = \tilde{k}_a S + \tilde{l}_a T + O(q)$ with some real constants $\tilde{k}_a$ and $\tilde{l}_a$, (iii) the Yukawa couplings $\tilde{\lambda} = (1+O(q))\tilde{\lambda}_0 \exp(aS+bT)$ where a and b are some real constants while the constant $\tilde{\lambda}_0$ is complex in general. In (iii), the constants a and b also can carry the omitted flavor indices [14]. Clearly the properties (i), (ii), (iii) ensure that the quantities of X_1 are all real up to corrections of $O(q)$.

A complex $\partial_I \ln(\tilde{W})$ would give a nonzero phase of the SUSY breaking order parameters F_I. In string theory, $\tilde{W}$ is induced by nonperturbative hidden sector dynamics. All renormalizable interactions of the hidden sector fields are determined by the hidden gauge kinetic functions f_h and the hidden Yukawa couplings $\tilde{\lambda}_h$ (up to the scaling due to non-minimal Kähler potential). Then $\tilde{W}$ would appear as a generic holomorphic function of the holomorphic quantities f_h and $\tilde{\lambda}_h$. Since it is nonperturbative in the hidden gauge coupling constant $1/\mathrm{Re}(f_h)$, $\tilde{W}$ is suppressed by some powers of e^{-f_h}. The forms of f_h

and $\tilde{\lambda}_h$ are restricted by $U(1)_S$ and $U(1)_T$ as in (i) and (ii). Then using the arguments involving anomaly free R-symmetries and dimensional analysis [15], $\tilde{W}$ can be written as [16]:

$$\tilde{W} = \sum_{n=1}^{N_W} W_n = \sum d_n(T) \exp(k_n S + l_n T) \tag{11}$$

where k_n and l_n are some *real* constants and $d_n(T) = \hat{d}_n(1 + O(q))$ for a complex constant $\hat{d}_n$.

Since the corrections less then $10^{-2} - 10^{-3}$ are essentially ignored in our approximation, $\tilde{W}$ includes only the terms such that $|W_n/W_1| \geq 10^{-2} - 10^{-3}$ where W_1 denotes the term with the largest vacuum value. Clearly the number of such terms, viz N_W, would depend on the details of the hidden sector, e.g. on the number of simple hidden gauge groups, the ratios of the dynamical mass scales, and also the Yukawa couplings. Let us briefly discuss the number of terms in $\tilde{W}$ for several simple cases. If the hidden sector contains a simple gauge group $\mathcal{G}_1$ whose dynamical mass scale Λ_1 is far above those of other groups, then $\tilde{W} \simeq W_1 \sim \Lambda_1^3$ where W_1 contains the gaugino condensation together with possible matter condensations. In the case that there exists another simple group $\mathcal{G}_2$ with Λ_2 comparable to Λ_1, $\tilde{W}$ contains at least two terms $W_{1,2} \sim \Lambda_{1,2}^3$. If the gaugino condensations are largely dominate over other possible contributions, e.g. matter condensations, one simply has $N_W = 2$ associated with the two gaugino condensations of $\mathcal{G}_1$ and $\mathcal{G}_2$. Even in the case that matter condensations become important, if the fields that transform nontrivially under $\mathcal{G}_1$ communicate weakly with those of $\mathcal{G}_2$, e.g. communicate only via nonrenormalizable interactions, one still has $N_W = 2$ but now W_1 and W_2 contain both the gaugino and matter

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condensations of the $\mathcal{G}_1$-sector and the $\mathcal{G}_2$-sector respectively.

As is well known, the case of $N_W = 1$ suffers from the runaway of the dilaton. Thus let us consider the next simple case of $N_W = 2$ which has been argued to be able to produce phemenologically interesting results [17]. In fact, most of $\tilde{W}$'s analyzed in the literatures have $N_W = 2$. It is obvious that $\partial_I \ln(W_{1,2})$ is real up to corrections of $O(q)$. However to have a real $X_2 = \partial_I \ln(\tilde{W})$, one still needs the relative phase $\arg(W_2/W_1)$ to be CP conserving. Interestingly enough, this can be achieved dynamically by the vacuum value of the model-independent axion $\mathrm{Im}(S)$. Using the standard scalar potential in supergravity, one easily find the following form of the axion-potential:

$$V_{\mathrm{axion}} = \Omega\left[\cos(\arg(W_2/W_1)) + O(q)\right], \tag{12}$$

where $\arg(W_2/W_1) = (k_2 - k_1)\mathrm{Im}(S) + \delta$, and Ω and δ are real functions which are *independent* of $\mathrm{Im}(S)$. Clearly minimizing this axion potential leads to a real value of W_2/W_1 up to $O(q)$, and thus a real value of $\partial_I \ln(\tilde{W})$ up to $O(q)$. Note that here $\delta = \arg(d_2/d_1) + (l_2 - l_1)\mathrm{Im}(T)$ is of order unity in general, but it is dynamically relaxed to a CP conserving value by the vacuum value of $\mathrm{Im}(S)$. This is quite similar to the Peccei-Quinn mechanism [18] relaxing θ in the axion solution to the strong CP problem. At any rate, now we find that X_1 and X_2 are real up to corrections of $O(q)$, and thus $\phi_{A,C} = O(q)$ if SUSY breaking is due to the auxiliary components of S and T with $N_W = 2$.

In the above, we have noted the dynamical relaxation of the relative phases between W_n's for $N_W = 2$. This mechanism can be easily generalized for $N_W > 2$. Suppose we have N_A axion-like fields $\vec{A} = (A_1, ..., A_{N_A})$ whose

nonderivative couplings are *mainly* given by $W_n \sim e^{i\vec{c}_n \cdot \vec{A}}$ due to the associated nonlinear PQ symmetries: $\vec{A} \rightarrow \vec{A} + \vec{\alpha}$. Here $\vec{c}_n$ and $\vec{\alpha}$ denote some real constant vectors. Note that we always have two such fields, $\text{Im}(S)$ and $\text{Im}(T)$, and thus $N_A \geq 2$. With more moduli, N_A would become larger. Let N denote the number of linearly independent vectors among $\{\vec{c}_n - \vec{c}_m\}$. Then for $N_W \leq N + 1$, all the relative phases $\arg(W_n/W_m)$ are relaxed to CP conserving values (of course up to corrections associated with the breakdown of the PQ symmetries). Note that $1 \leq N \leq N_A$ in general, but it is quite conceivable to have $N = N_A$.

The discussion of the remained phase ϕ_B is more model-dependent since presently there is no definite theory for the μ-term [19]. Here we consider three simple scenarios in which one of $\mu_{1,2,3}$ dominates over the other two by more than a factor of 10^2 to 10^3. In the first case that $\mu_1 = \lambda \langle N \rangle$ dominates, ϕ_B simply corresponds to ϕ_A and thus is of $O(q)$. In the second case that $\mu_2 = (m_{3/2} - \bar{F}_I \bar{\partial}_I)Y$ dominates, ϕ_B would receive additional contribution from X_4. Orbifold compactifications give $Y = 0$ [10] and thus they do not correspond to this case. It has been pointed out that for $(2,2)$ Calabi-Yao compactifications Y is related to some Yukawa couplings [10] by world sheet Ward identities. One then has $X_4 = O(q)$ [5], implying $\phi_B = O(q)$. In the third case that μ_3 dominates, ϕ_B would receive a contribution from $X_3 = \partial_I \ln(\tilde{\mu})$. Since $\tilde{\mu}$ is due to nonperturbative effects, it can be written as $\tilde{\mu} = \sum \tilde{\mu}_n$ where $\tilde{\mu}_n = (1 + O(q))z_n \exp(x_n S + y_n T)$. Here x_n and y_n are some real constants while z_n is a complex constant. Again $\partial_I \ln(\tilde{\mu}_n)$ is real up to $O(q)$, however to have real $\partial_I \ln(\tilde{\mu})$ one still needs the relative phases of $\tilde{\mu}_n$ to be CP conserving. For the induced superpotential $\tilde{W}$, the relative

phases of W_n could be relaxed to CP conserving values by the vacuum values of the axion-like fields. For $\tilde{\mu}$, we do not have any such mechanism. Of course, if there is just a single term in $\tilde{\mu}$, $\partial_I \ln(\tilde{\mu})$ is real up to $O(q)$ and thus $\phi_B = O(q)$. If there are more than one term, e.g. $\tilde{\mu}_{1,2}$ such that $\tilde{\mu}_1 > \tilde{\mu}_2$, both $\arg(\partial_I \ln(\tilde{\mu}))$ and ϕ_B would be of $O(\tilde{\mu}_2/\tilde{\mu}_1)$ in general.

So far, our discussion has been restricted to the case with S and T only. It is in fact straightforward to extend the analysis to the case with more moduli. Let us suppose an additional modulus M and define the corresponding PQ symmetry $U(1)_M : M \to M + i\alpha_M$. This additional modulus can affect our previous analysis by two ways. First of all, it can directly affect the SUSY phases by participating in SUSY breaking, i.e. by having a nonzero auxiliary component F_M. Secondly it can affect F_I ($I = S, T$) via the wave function mixing with S and T. Let q_M denote a factor characterizing the size of $U(1)_M$-breaking corrections. Then including M in the analysis, it is easy to see that the SUSY phases receive additional contributions which are of the order of either $q_M F_M / m_{3/2}$ or $q_M \bar{\partial}_I \partial_M \tilde{K} / \bar{\partial}_I \partial_I \tilde{K}$. Thus even in the case with more moduli, the SUSY phases are suppressed by a factor governing the breakdown of the PQ symmetries nonlinearly realized for the moduli that participate in SUSY breaking.

What would be the typical size of the PQ symmetry breakings? For a Kähler class modulus M_K that is associated with the deformation of the Kähler class of the internal space, e.g. the overall modulus T, the pseudoscalar component $\text{Im}(M_K)$ comes from the zero modes of the antisymmetric tensor field. Then the corresponding PQ symmetry is broken only by world sheet instantons [6], leading to $q_{M_K} = e^{-2\pi M_K}$ which can be small

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enough to give the SUSY phases less than $10^{-2} - 10^{-3}$ for a moderately large value of $\text{Re}(M_K)$. For another type of moduli, the complex structure moduli M_C that is associated with the deformation of the complex structure, the size of q_{M_C} is somewhat model-dependent. For orbifold compactifications, q_{M_C} is exponentially small due to the modular symmetry $SL(2,Z)$ [20]. However for Calabi-Yao cases, q_{M_C} can be of order unity even at the leading order approximation [6]. As a result, to achieve small SUSY phases in Calabi-Yao compactification, one needs to assume that the complex structure moduli give negligible contribution to SUSY breaking, viz $F_{M_C}/m_{3/2} \leq 10^{-2} - 10^{-3}$, and also have small wave function mixings with the Kähler moduli M_K, viz $\bar{\partial}_{M_C} \partial_{M_K} \tilde{K} / \bar{\partial}_{M_K} \partial_{M_K} \tilde{K} \leq 10^{-2} - 10^{-3}$, or needs some assumption on CP violation.

Barring the dynamical relaxation of the relative phases in $\tilde{W}$, it is a general conclusion in string-inspired supergravity that the SUSY phases $\phi_{A,C}$ are suppressed by a factor governing the breakdown of the PQ symmetries nonlinearly realized for the moduli that participate in SUSY breaking. A similar suppression can occur also for ϕ_B if the μ-term is dominated by either μ_1 or μ_2. In many cases, particularly for generic Kähler class moduli, the PQ symmetry breaking factors are exponentially small for a moderately large values of the moduli. Then one would achieve small SUSY phases, i.e. $\phi_{A,B,C} \leq 10^{-2} - 10^{-3}$, in a quite natural way in string-inspired supergravity models. In regard to the dynamical relaxation mechanism, we have argued that it can take place for $N_W \leq N_A + 1$ where N_A denotes the number of the axion-like fields which is essentially the same as the number of moduli (including S) with good PQ symmetries. Most of $\tilde{W}$'s analyized in the

literatures have $N_W = 2$ for which the relaxation always occurs.

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